Test one in math 611 7/4/2012

1- a) Show that the space l^p is complete.

2- Show that the set X of all continuous real valued functions on $[0,1]$ is not complete.

3- a) Show that the space s is metric

b) Show that in the space s we have $x_n \to x$ iff $\zeta_j^{(n)} \to \zeta_j$ for all j=1,2,....... where $x_n = (\zeta_j^{(n)})$, x=(ζ_j)

c) Show that the space s is complete.

4- Let T be a bounded linear operator from a normed space X onto a normed space Y . If there a positive b such that $\|Tx\| \geq b\|x\|$. Show that $T^{-1} : Y \to X$ is bounded.

5-a) Let T:C$[0,1] \to C[0,1]$ be defined by y(t)=$\int_0^t x(\tau)d\tau$. find R(T) and $T^{-1} : R(T) \to C[0,1]$ is T^{-1} linear and bounded?

b) Define a linear functional f on C$[a,b]$ as f(x)=$\int_a^b x(t)dt$ x$\in C[a,b]$ prove that $\|f\| = b - a$

6- State and prove Hann- Banach theorem.

7- Let X,Y be normed spaces and $T_n : X \to Y$ be bounded linear operators , show that if $T_n \to T$ implies for every $\in > 0, \exists N$ such that n$\geq N$ and x is any closed ball, we have $\|T_n x - Tx\| < \in$.

8- Let T be abounded linear operator prove that a) if $x_n \to x$ then $Tx_n \to Tx$

b) N(T) null space is closed.

9) If T is a bounded linear operator shoe that for any x$\in D(T)$ s.t. $\|x\| < 1$ then $\|Tx\| < \|T\|$

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